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LETTERS TO THE EDITOR

VIBRATIONS OF AN ANNULAR CIRCULAR PLATE OF POLAR ANISOTROPY WITH ONE EDGE SUPPORTED, THE OTHER ONE FREE AND AN INTERMEDIATE CONCENTRIC CIRCULAR SUPPORT

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1. INTRODUCTION

The present study constitutes an extension of the problem treated by Vega *et al.* [1], where an exact solution was obtained for the isotropic structural configuration. An approximate solution is obtained in the present paper in the case where the constitutive relations are polarly anisotropic[†] (see Figure 1). The problem is solved by means of the optimized Rayleigh–Ritz using several optimization parameters [3] and polynomial co-ordinate functions.

2. APPROXIMATE SOLUTION

When the system executes transverse, normal modes of vibration the problem is governed by the functional [2]

$$J(W) = \frac{1}{2}\int_a^b \int_0^{2\pi} \left\{ D_r \left(\frac{\mathrm{d}^2 W}{\mathrm{d}r^2}\right)^2 + D_\theta \left(\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2 + 2v_\theta D_r \left[\left(\frac{\mathrm{d}^2 W}{\mathrm{d}r^2}\right)\left(\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)\right]\right\} r\,\mathrm{d}r\,\mathrm{d}\theta - \frac{\rho h}{2}\omega^2 \int_a^b \int_0^{2\pi} W^2 r\,\mathrm{d}r\,\mathrm{d}\theta \tag{1}$$

subjected to appropriate boundary conditions.

Regarding the natural boundary conditions at the free edge it was decided to run the following numerical experiments: (A) satisfying the condition of nulle radial moment at the free edge and (B) not satisfying it. It must be clarified that the condition of nulle Kelvin–Kirchhoff force at the free edge was disregarded in both experiments.

In correspondence with the first set of experiments the boundary conditions are:

Case I (free inside and clamped outside, Figure 1 (I)):

$$W(c) = 0, \qquad W(b) = 0, \qquad W'(b) = 0, \qquad W''(a) + \frac{v_\theta}{a} W'(a) = 0. \tag{2a–d}$$

[†] Also called polarly orthotropic [2].

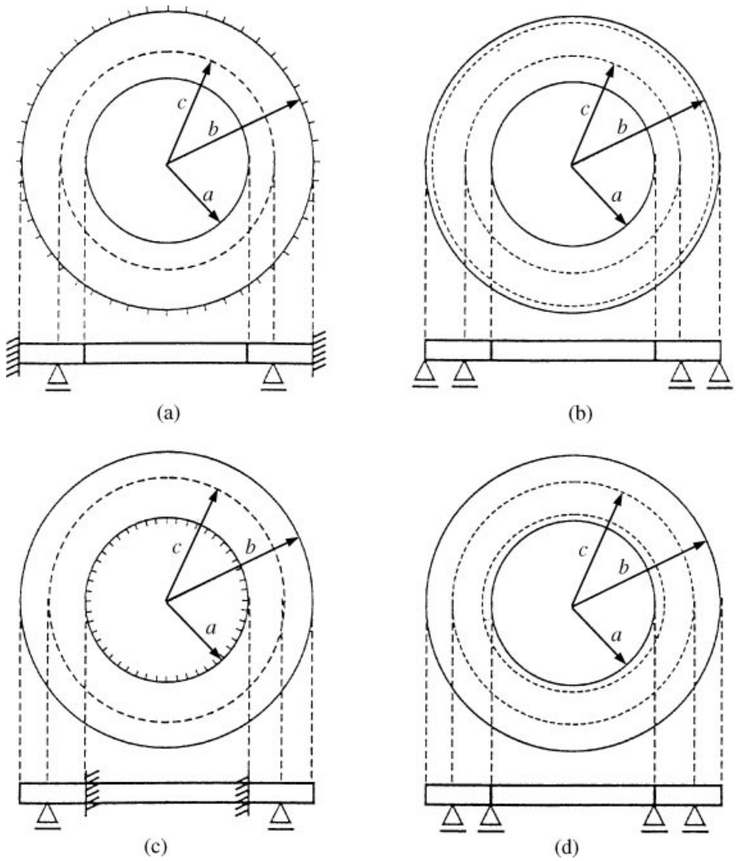

Figure 1. Vibrating systems under study: (a) Case I, (b) Case II, (c) Case III, (d) Case IV.

Case II (free inside and simply supported outside, Figure 1 (II):

$$W(c) = 0, \qquad W(b) = 0, \qquad W''(b) + \frac{v_\theta}{b} W'(b) = 0, \qquad W''(a) + \frac{v_\theta}{a} W'(a) = 0. \qquad \text{(3a–d)}$$

Case III (free outside and clamped inside, Figure 1 (III)):

$$W(c) = 0, \qquad W(a) = 0, \qquad W'(a) = 0, \qquad W''(b) + \frac{v_\theta}{b} W'(b) = 0. \qquad \text{(4a–d)}$$

Case IV (free outside and simply supported inside, Figure 1 (IV)):

$$W(c) = 0, \qquad W(a) = 0, \qquad W''(a) + \frac{v_\theta}{a} W'(a) = 0, \qquad W''(b) + \frac{v_\theta}{b} W'(b) = 0. \qquad \text{(5a–d)}$$

The following expression with four exponential optimization parameters has been used:

$$W(r) \cong W_z(r) = C_1(\alpha_p r^p + \alpha_q r^q + \alpha_s r^s \div \alpha_u r^u + 1)$$
$$+ C_2(\beta_p r^{p+1} + \beta_q r^{q+1} + \beta_s r^{s+1} + \beta_u r^{u+1} + 1), \qquad (6)$$

where the α's and β's are determined substituting each co-ordinate function in the boundary conditions and p, q, s, and u are Rayleigh's optimization parameters.

When performing the second set of experiments, $W(r)$ has been approximated by means of

$$W(r) \cong W_a(r) = C_1(\alpha_p r^p + \alpha_q r^q + \alpha_s r^s + 1) + C_2(\beta_p r^{p+1} + \beta_q r^{q+1} + \beta_s r^{s+1} + 1), \quad (7)$$

where now the nulle radial bending moment requirement is not satisfied.

Substituting the approximating expressions in the governing functional and making use of the Rayleigh–Ritz classical procedure one obtains the fundamental frequency coefficient Ω_1, where

$$\Omega_1 = \sqrt{\frac{\rho h}{D_r}}\,\omega_1 b^2 = \Omega_1(p, q, s, u) \quad (8)$$

TABLE 1

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support

			Free inside and clamped outside (Figure 1, Case I)				Free inside and simply supported outside (Figure 1, Case II)			
$\frac{a}{b}$	$\frac{c}{b}$	$\frac{D_\theta}{D_r}$	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0·1	0·2	0·25	26·58	*25·63*	25·96	*25·74*	16·20	15·53	15·73	*15·67*
0·1	0·2	1	30·52	*28·56*	28·96	*28·68*	19·50	*18·23*	18·44	*18·37*
0·1	0·2	2	33·81	*31·05*	31·45	*31·11*	22·19	*20·50*	20·68	*20·58*
0·1	0·5	0·25	25.52	*23·52*	24·18	*23·75*	22·47	*21·10*	21·41	*21·25*
0·1	0·5	1	34·87	*31·02*	31·93	*31·38*	30·00	*27·70*	28·19	*28·06*
0·1	0·5	2	41·65	*36·90*	*37·71*	*36·90*	34·72	*32·60*	33·02	*32·67*
0·1	0·9	0·25	9·373	*9·110*	10·70	*9·180*	9·110	*9·000*	10·24	*9·000*
0·1	0·9	1	12·31	*11·97*	14·29	*12·01*	11·94	*11·80*	13·67	*11·80*
0·1	0·9	2	14·23	*13·80*	16·83	*13·81*	13·80	*13·56*	16·09	*13·58*
0·4	0·5	0·25	62·40	*61·40*	61·96	*61·69*	39·18	*38·45*	38·97	*38·84*
0·4	0·5	1	63·85	*62·74*	63·33	*63·05*	40·56	*40·00*	40·33	*40·17*
0·4	0·5	2	65·65	*64·27*	65·05	64·73	42·27	*41·61*	42·02	*41·82*
0·4	0·9	0·25	16·12	*15·78*	17·75	*15·79*	15·52	*15·37*	16·74	*15·38*
0·4	0·9	1	17·68	*17·34*	19·58	*17·34*	17·06	*16·92*	18·54	*16·93*
0·4	0·9	2	19·46	*19·09*	21·72	*19·09*	18·82	*18·67*	20·61	*18·70*
0·8	0·9	0·25	263·9	*256·7*	262·6	*258·1*	240·3	*236·5*	239·1	*237·3*
0·8	0·9	1	264·7	*257·4*	263·4	*259·0*	241·1	*237·4*	239·9	238·2
0·8	0·9	2	265·8	*258·3*	*264·5*	*260·1*	242·3	*238·5*	241·1	*239·4*

Note: Effect of different optimization schemes.
(1) Without optimization ($p = 4$, $q = 3$, $s = 2$ and $u = 1$) [equation (6)].
(2) Optimizing with respect to p, q, s and u [equation (6)]
(3) Without optimization ($p = 3$, $q = 2$ and $s = 1$) [equation (7)].
(4) Optimizing with respect to p, q and s [equation (7)].

in the case of the set of experiments (A), and

$$\Omega_1 = \sqrt{\frac{\rho h}{D_r}}\,\omega_1 b^2 = \Omega_1(p, q, s) \tag{9}$$

for experiments (B).

By numerical and simultaneous minimization parameters one is able to optimize the fundamental frequency coefficient as a function of the intervening geometric and mechanical parameters of the system.

3. NUMERICAL RESULTS

The numerical determinations have been performed for $v_0 = 0{\cdot}3$ and several values of a/b, c/b, and D_θ/D_r.

Tables 1 and 2 illustrate in a very clear fashion the results of (1) satisfying one of the natural boundary conditions and (2) the optimization scheme. When (1) and (2) are combined the results are lower and, accordingly, more accurate.

TABLE 2

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support

			Free outside and clamped inside (Figure 1, Case III)				Free outside and simply supported inside (Figure 1, Case IV)			
$\frac{a}{b}$	$\frac{c}{b}$	$\frac{D_\theta}{D_r}$	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0·1	0·2	0·25	4·722	*3·977*	5·990	*4·130*	4·461	*3·891*	5·610	*3·960*
0·1	0·2	1	5·615	*4·937*	6·840	*5·086*	5·374	*4·867*	6·470	*4·930*
0·1	0·2	2	6·612	*5·981*	7·820	*6·130*	6·385	*5·918*	7·470	*5·993*
0·1	0·5	0·25	8·639	*8·628*	9·600	*8·820*	8·025	*7·995*	8·370	*8·000*
0·1	0·5	1	9·554	*9·549*	10·46	*9·770*	9·198	*9·102*	9·430	*9·112*
0·1	0·5	2	10·65	*10·65*	11·51	*10·89*	10·55	*10·34*	10·67	*10·39*
0·1	0·9	0·25	21·05	*20·88*	22·77	*21·06*	15·19	*15·10*	15·21	*15·14*
0·1	0·9	1	22·53	*22·39*	24·03	*22·57*	18·42	*18·04*	18·33	*18·09*
0·1	0·9	2	24·36	*24·23*	25·61	*24·41*	21·80	*20·99*	21·80	*21·15*
0·4	0·5	0·25	12·00	*11·25*	14·48	*11·66*	11·35	*10·95*	13·43	*11·09*
0·4	0·5	1	12·72	*11·97*	15·21	*12·40*	12·08	*11·62*	14·17	*11·84*
0·4	0·5	2	13·61	*12·86*	16·13	*13·32*	12·99	*12·49*	15·10	*12·77*
0·4	0·9	0·25	56·13	*55·93*	58·01	*56·51*	38·12	*38·01*	39·19	*38·03*
0·4	0·9	1	56·96	*56·75*	58·86	*57·34*	39·30	*39·18*	40·33	*39·19*
0·4	0·9	2	58·04	*57·82*	59·96	*58·46*	40·83	*40·68*	41.79	*40·68*
0·8	0·9	0·25	247·2	*241·6*	247·4	*241·8*	225·2	*221·1*	224·4	*221.1*
0·8	0·9	1	247·8	*242·3*	248·1	*242·4*	225·9	*221·8*	225·0	*221·8*
0·8	0·9	2	248·7	*243·2*	248·9	*243·3*	226·8	*222·8*	225·9	*222·8*

Note: Effect of different optimization schemes.
(1) Without optimization ($p = 4$, $q = 3$, $s = 2$ and $u = 1$) [equation (6)].
(2) Optimizing with respect to p, q, s and u [equation (6)]
(3) Without optimization ($p = 3$, $q = 2$ and $s = 1$) [equation (7)].
(4) Optimizing with respect to p, q and s [equation (7)].

Tables 3, 4, 5 and 6 correspond to cases, I, II, III and IV of Figure 1 respectively. The fundamental eigenvalues have been evaluated taking into account the nulle radial bending moment condition at the free edge and optimizing with respect to p, q, s and u.

In the case of isotropic structural systems one has $D_\theta/D_r = 1$ and the present, approximate results are in very good engineering agreement with the eigenvalues obtained following the exact approach [1]. It is reasonable to expect then that for $D_\theta/D_r \neq 1$ the

TABLE 3

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support, free inside and clamped outside (Figure 1, Case I)

		D_θ/D_r							
a/b	c/b	0·25	0·50	0·75	1	1·25	1·50	1·75	2
0·1	0·2	25·63	26·65	27·74	28·56	29·33	30·01	30·56	31·05
0·1	0·3	31·83	33·17	34·52	35·49	36·23	37·02	37·68	38·28
0·1	0·4	31·64	34·91	37·32	39·25	40·84	42·15	43·29	44·24
0·1	0·5	23·52	26·60	28·97	31·02	32·81	34·21	35·73	36·90
0·1	0·6	16·79	19·83	21·49	23·02	24·23	25·29	26·25	27·14
0·1	0·7	13·59	15·44	16·78	17·85	18·73	19·49	20·16	20·77
0·1	0·8	11·01	12·54	13·58	14·42	15·14	15·72	16·24	16·71
0·1	0·9	9·110	10·52	11·37	11·97	12·60	13·10	13·51	13·80
0·2	0·3	32·68	33·42	34·09	34·72	35·31	35·85	36·40	36·90
0·2	0·4	39·08	40·43	41·61	42·67	43·61	44·49	45·27	45·99
0·2	0·5	30·84	32·75	34·51	36·10	37·58	38·96	40·25	41·46
0·2	0·6	21·40	22·99	24·41	25·67	26·84	27·93	28·94	29·87
0·2	0·7	15·75	17·05	18·24	19·19	20·14	20·96	21·71	22·41
0·2	0·8	12·31	13·40	14·37	15·11	15·91	16·56	17·15	17·69
0·2	0·9	9·980	10·96	11·75	12·39	13·05	13·58	14·06	14·49
0·3	0·4	43·67	44·24	44·73	45·25	45·75	46·21	46·66	47·13
0·3	0·5	47·19	48·36	49·43	50·44	51·46	52·39	53·30	54·18
0·3	0·6	31·21	32·36	33·44	34·46	35·48	36·41	37·35	38·21
0·3	0·7	21·05	22·00	22·89	23·72	24·51	25·25	25·95	26·62
0·3	0·8	15·32	16·30	17·04	17·66	18·37	18·97	19·54	20·08
0·3	0·9	12·00	12·72	13·37	13·96	14·50	15·00	15·47	15·91
0·4	0·5	61·40	61·84	62·31	62·74	63·16	63·56	63·96	64·27
0·4	0·6	54·59	55·45	56·29	57·06	57·94	58·74	59·42	60·16
0·4	0·7	32·43	33·12	33·82	34·49	35·17	35·81	36·44	37·05
0·4	0·8	21·63	22·24	22·82	23·35	23·92	24·45	24·96	25·45
0·4	0·9	15·78	16·34	16·85	17·34	17·81	18·26	18·69	19·09
0·5	0·6	91·37	91·81	92·21	92·62	93·02	93·41	93·81	94·20
0·5	0·7	61·00	61·60	62·20	62·79	63·37	63·94	64·51	65·08
0·5	0·8	34·39	34·89	35·35	35·82	36·28	36·73	37·17	38·06
0·5	0·9	22·84	23·27	23·69	24·10	24·49	24·88	25·25	25·62
0·6	0·7	140·9	141·3	141·8	142·2	142·7	143·6	144·0	144·4
0·6	0·8	67·60	68·01	68·42	68·82	70·30	71·05	71·45	71·85
0·6	0·9	37·65	38·00	38·35	38·64	39·02	39·35	39·68	40·00
0·7	0·8	201·3	201·7	202·1	202·4	202·8	203·2	203·5	203·8
0·7	0·9	77·05	77·34	77·53	77·73	78·02	78·30	78·59	78·87
0·8	0·9	256·7	256·9	257·1	257·4	257·6	257·8	258·1	258·3

Note: The values have been determined by optimizing with respect to p, q, s and u [equation (6)].

Table 4

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support, free inside and simply supported outside (Figure 1, Case II)

		D_θ/D_r							
a/b	c/b	0·25	0·50	0·75	1	1·25	1·50	1·75	2
0·1	0·2	15·53	16·51	17·30	18·23	19·01	19·64	20·19	20·50
0·1	0·3	20·12	21·34	22·25	22·95	23·70	24·30	24·85	25·35
0·1	0·4	23·86	25·83	27·30	28·37	29·23	29·96	30·67	31·25
0·1	0·5	21·10	23·91	26·04	27·70	29·24	30·52	31·62	32·60
0·1	0·6	16·36	18·68	20·40	21·79	22·96	24·40	24·93	25·80
0·1	0·7	12·96	14·78	16·17	17·14	18·00	18·74	19·40	20·01
0·1	0·8	10·64	12·14	13·18	13·99	14·67	15·24	15·75	16·21
0·1	0·9	9·000	10·27	11·22	11·80	12·34	12·80	13·20	13·56
0·2	0·3	20·00	20·71	21·45	22·04	22·61	23·14	23·64	24·10
0·2	0·4	25·91	26·90	27·78	28·54	29·29	29·92	30·50	31·05
0·2	0·5	25·83	27·46	29·37	30·73	31·96	33·08	34·10	35·04
0·2	0·6	19·74	21·28	22·74	23·98	25·12	26·19	27·15	28·06
0·2	0·7	14·91	16·25	17·35	18·34	19·23	20·03	20·77	21·46
0·2	0·8	11·83	12·94	13·87	14·63	15·30	16·03	16·60	17·13
0·2	0·9	9·757	10·71	11·51	12·17	12·76	13·29	13·75	14·18
0·3	0·4	27·13	27·67	28·18	28·68	29·15	29·61	30·07	30·49
0·3	0·5	34·17	35·05	35·89	36·67	37·41	38·11	38·78	39·41
0·3	0·6	27·94	29·08	30·15	31·16	32·12	33·04	33·93	34·78
0·3	0·7	19·73	20·70	21·56	22·46	23·23	23·98	24·63	25·31
0·3	0·8	14·79	15·60	16·35	16·98	17·68	18·28	18·84	19·37
0·3	0·9	11·74	12·45	13·08	13·64	14·21	14·71	15·17	15·61
0·4	0·5	38·45	39·15	39·59	40·00	40·42	40·82	41·23	41·61
0·4	0·6	44·13	44·92	45·70	46·43	47·17	47·87	48·57	49·25
0·4	0·7	29·81	30·54	31·26	31·96	32·64	33·30	33·93	34·59
0·4	0·8	20·50	21·12	21·71	22·27	22·82	23·35	23·87	24·41
0·4	0·9	15·37	15·92	16·43	16·92	17·39	17·83	18·27	18·67
0·5	0·6	59·17	59·57	59·96	60·35	60·74	61·11	61·48	61·85
0·5	0·7	53·54	54·16	54·77	55·37	55·96	56·55	57·12	57·69
0·5	0·8	32·27	32·77	33·27	33·74	34·21	34·65	35·12	35·56
0·5	0·9	22·12	22·55	22·97	23·37	23·77	24·16	24·54	24·91
0·6	0·7	98·12	98·51	98·90	99·28	99·67	100·0	100·4	100·8
0·6	0·8	61·98	62·41	62·83	63·25	63·68	64·04	64·51	64·92
0·6	0·9	36·16	36·51	36·83	37·12	37·51	37·87	38·20	38·53
0·7	0·8	165·4	165·8	166·1	166·6	167·0	167·4	167·7	168·0
0·7	0·9	72·54	72·83	73·13	73·38	73·71	74·01	74·30	74·59
0·8	0·9	236·5	236·8	237·1	237·4	237·7	238·0	238·3	238·5

Note: The values have been determined by optimizing with respect to p, q, s and u [equation (6)].

fundamental frequency coefficients determined using the optimized Rayleigh–Ritz method and polynomial co-ordinate functions will possess adequate accuracy from a practical viewpoint.

It is interesting to point out that if one compares columns (1) and (3) of Tables 1 and 2 the results of column 1 are, in some cases, slightly higher than those corresponding to column 3.

TABLE 5

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support, clamped inside and free outside (Figure 1, Case III)

		D_θ/D_r							
a/b	c/b	0·25	0·50	0·75	1	1·25	1·50	1·75	2
0·1	0·2	3·937	4·3207	4·629	4·937	5·218	5·484	5·738	5·987
0·1	0·3	5·029	5·348	5·649	5·934	6·206	6·466	6·725	6·956
0·1	0·4	6·462	6·774	7·071	7·357	7·631	7·895	8·150	8·397
0·1	0·5	8·628	8·947	9·253	9·549	9·837	10·11	10·38	10·65
0·1	0·6	12·12	12·48	12·81	13·14	13·46	13·77	14·08	14·37
0·1	0·7	17·94	18·37	18·79	19·19	19·59	19·97	20·35	20·71
0·1	0·8	23·49	24·07	24·63	25·17	25·71	26·22	26·73	27·23
0·1	0·9	20·88	21·41	21·92	22·39	22·88	23·35	23·80	24·23
0·2	0·3	5·416	5·716	6·001	6·272	6·531	6·780	7·010	7·250
0·2	0·4	7·046	7·330	7·603	7·866	8·120	8·365	8·604	8·835
0·2	0·5	9·422	9·703	9·977	10·24	10·50	10·75	11·00	11·24
0·2	0·6	13·30	13·59	13·88	14·16	14·44	14·71	14·98	15·24
0·2	0·7	20·20	20·54	20·87	21·13	21·52	21·83	22·14	22·45
0·2	0·8	29·97	30·40	30·81	31·23	31·64	32·04	32·45	32·84
0·2	0·9	28·25	28·62	29·00	29·36	29·72	30·08	30·42	30·76
0·3	0·4	7·629	7·898	8·157	8·408	8·651	8·866	9·116	9·338
0·3	0·5	10·28	10·54	10·80	11·04	11·28	11·52	11·75	11·98
0·3	0·6	14·57	14·83	15·09	15·34	15·59	15·84	16·08	16·32
0·3	0·7	22·44	22·73	23·01	23·29	23·56	23·84	24·11	24·38
0·3	0·8	36·89	37·24	37·60	37·92	38·28	38·62	38·96	39·30
0·3	0·9	39·02	39·32	39·64	39·94	40·25	40·54	40·83	41·12
0·4	0·5	11·25	11·49	11·73	11·97	12·20	12·42	12·63	12·86
0·4	0·6	16·05	16·29	16·53	16·76	17·00	17·22	17·44	17·67
0·4	0·7	24·89	25·15	25·40	25·64	25·89	26·14	26·38	26·62
0·4	0·8	43·75	44·04	44·34	44·64	44·93	45·23	45·52	45·89
0·4	0·9	55·93	56·23	56·50	56·75	57·05	57·31	57·57	57·82
0·5	0·6	17·79	18·01	18·23	18·46	18·67	18·89	19·09	19·26
0·5	0·7	27·86	28·08	28·31	28·54	28·76	28·99	29·21	29·43
0·5	0·8	50·69	50·93	51·14	51·38	51·64	51·89	52·15	52·41
0·5	0·9	84·05	84·31	84·57	84·83	85·10	85·35	85·61	85·86
0·6	0·7	31·52	31·71	31·94	32·15	32·36	32·57	32·77	32·98
0·6	0·8	58·39	58·62	58·85	59·03	59·32	59·55	59·78	60·00
0·6	0·9	129·3	129·6	129·8	130·1	130·4	130·7	131·0	131·3
0·7	0·8	68·72	68·93	69·14	69·34	69·55	69·76	69·93	70·13
0·7	0·9	183·8	184·1	184·4	184·6	184·9	185·2	185·5	185·7
0·8	0·9	241·6	241·8	242·1	242·3	242·6	242·8	243·0	243·2

Note: The values have been determined by optimizing with respect to p, q, s and u [equation (6)].

One concludes that satisfying an additional boundary condition does not necessarily improve the results, at least when a small number of co-ordinate functions is used. Certainly, when the optimization scheme is employed, the results show considerable improvement when the nulle radial bending moment condition is applied; see columns (2) and (4) of both the tables.

TABLE 6

Fundamental frequency coefficients for an annular plate of polar orthotropy, with intermediate support, simply supported inside and free outside (Figure 1, Case IV)

		D_θ/D_r							
a/b	c/b	0·25	0·50	0·75	1	1·25	1·50	1·75	2
0·1	0·2	3·891	4·274	4·595	4·867	5·178	5·455	5·700	5·918
0·1	0·3	4·785	5·125	5·443	5·743	6·028	6·300	6·559	6·808
0·1	0·4	6·069	6·421	6·755	7·063	7·365	7·651	7·926	8·186
0·1	0·5	8·000	8·391	8·759	9·103	9·433	9·750	10·05	10·34
0·1	0·6	11·02	11·51	11·96	12·38	12·78	13·15	13·51	13·86
0·1	0·7	15·49	16·23	16·90	17·52	18·10	18·63	19·13	19·61
0·1	0·8	17·76	18·90	19·93	20·87	21·74	22·55	23·32	24·04
0·1	0·9	15·11	16·19	17·16	18·05	18·86	19·62	20·33	20·99
0·2	0·3	5·355	5·620	5·887	6·150	6·409	6·660	6·902	7·130
0·2	0·4	6·687	6·985	7·266	7·544	7·805	8·061	8·308	8·548
0·2	0·5	8·776	9·094	9·384	9·474	9·955	10·22	10·48	10·74
0·2	0·6	12·18	12·53	12·87	13·20	13·52	13·83	14·14	14·44
0·2	0·7	17·76	18·23	18·67	19·11	19·53	19·94	20·33	20·71
0·2	0·8	22·76	23·45	24·11	24·75	25·37	25·97	26·56	27·12
0·2	0·9	19·65	20·31	20·95	21·56	22·15	22·72	23·27	23·81
0·3	0·4	7·403	7·676	8·046	8·193	8·439	8·677	8·908	9·133
0·3	0·5	9·755	10·02	10·29	10·54	10·79	11·04	11·28	11·51
0·3	0·6	13·50	13·79	14·08	14·32	14·64	14·90	15·17	15·43
0·3	0·7	20·15	20·51	20·85	21·18	21·52	21·84	22·17	22·48
0·3	0·8	29·30	29·80	30·28	30·76	31·23	31·69	32·14	32·59
0·3	0·9	26·68	27·16	27·64	28·10	28·56	29·01	29·45	29·88
0·4	0·5	10·95	11·18	11·40	11·62	11·84	12·06	12·28	12·49
0·4	0·6	15·10	15·36	15·61	15·85	16·10	16·34	16·57	16·81
0·4	0·7	22·77	23·06	23·34	23·60	23·91	24·17	24·45	24·71
0·4	0·8	36·91	37·29	37·67	38·04	38·41	38·78	39·14	39·50
0·4	0·9	38·03	38·42	38·81	39·19	39·57	39·96	40·32	40·68
0·5	0·6	17·18	17·39	17·60	17·81	18·02	18·23	18·44	18·65
0·5	0·7	25·94	26·19	26·43	26·56	26·92	27·15	27·39	27·63
0·5	0·8	44·85	45·14	45·44	45·71	46·04	46·34	46·64	46·93
0·5	0·9	57·79	58·13	58·47	58·78	59·09	59·47	59·70	60·11
0·6	0·7	30·14	30·35	30·56	30·78	30·99	31·20	31·41	31·60
0·6	0·8	53·41	53·66	53·91	54·15	54·42	54·67	54·92	55·17
0·6	0·9	94·25	94·57	94·88	95·20	95·51	95·82	96·13	96·43
0·7	0·8	64·45	64·98	65·22	65·41	65·67	65·84	66·08	66·32
0·7	0·9	153·8	154·1	154·4	154·7	155·0	155·3	155·6	155·9
0·8	0·9	221·1	221·3	221·6	221·8	222·1	222·3	222·6	222·8

Note: The values have been determined by optimizing with respect to p, q, s and u [equation (6)].

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